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RELATIVISTIC SPONTANEOUS LOCALIZATION: A PROPOSAL

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Abstract — A new proposal for a Lorentz–invariant spontaneous localization theory is presented. It is based on the choice of a suitable set of macroscopic quantities to be stochastically induced to have definite values. Such macroscopic quantities have the meaning of a time–integrated amount of a microscopically defined quantity called stuff related to the presence of massive particles.

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1. Introduction

Either the wave function, as given by the Schrödinger equation is not everything, or it is not right. John Bell[1] expressed with this cathegoric sentence his point of view about the problem of quantum measurement. A possible way out from this dylemma consists in assuming that the wave function describes statistical ensembles rather than individual systems and resorting to the practical impossibility (decoherence) of detecting interference between the different macroscopically distinguishable terms appearing in the wave function after a measurement. The latter interpretative attitude[2] works to a considerable extent, but it cannot avoid certain typical inconsistencies[3,4] which arise from forgoing any tool apt to describe the result of an individual measurement. If the outcome of an individual measurement is to have a counterpart in the description of the system after the measurement, then one is led back to Bell's dylemma.

That the wave function is not everything means that there are additional variables which, together with the wave function, constitute the state of the system. A theory of this kind is Bohm's pilot–wave formulation of quantum mechanics,[5] where the additional variables are identified with the configuration of the system. Also the branch labels used to describe the system in the history approach to quantum mechanics[6−8] can be considered as additional variables.

That the wave function is not right means that a modification of the Schrödinger equation is to be accepted. As a matter of fact, the reduction principle of the standard formulation itself is a modification of the Schrödinger equation, a modification which allows to interpret the wave function as describing an individual system. The reduction principle, however, can be formulated only accompanied by the ambiguous distinction between quantum systems and measuring apparatus. Reduction theories, instead, describe reduction by a definite, mathematically precise correction to the Schrödinger equation, the corrected equation being supposed to be valid in any circumstance. The correction must rapidly reduce superpositions of macroscopically distinguishable states and, nevertheless, have practically unobservable consequences in all ordinary situations. Reduction theories can be considered as quantitative versions of the standard reduction principle. They are necessarily stochastic and nonlinear, just because the reduction principle is such.

In reduction theories[9−13] a stochastic process is introduced which induces the quantities belonging to a suitable set to have definite values. Such quantities are defined quantum mechanically but have a macroscopic character. They are always related to position, so that the result of the process is a localization of macroscopic objects. The localization is spontaneous in the sense that we postulate its existence at the level of the fundamental equation of quantum mechanics, without attempting to find its origin in terms of a level of description going beyond that. Successful reduction theories are all based on a spontaneous localization process which becomes effective only when the macroscopic level is reached. The reduction of the wave function of a measured microscopic quantum mechanical system takes place via the micro–macro correlations settled by the measuring device.

As it will be described in sect. 3, the definition of the macroscopic quantities involves an integration over a tiny space region, of a spherical shape for reasons of invariance of the theory. Such spherical regions cannot reduce to a point, because the quantities would lose their macroscopic character, exhibiting fluctuations related to the microscopic structure of the system. Furthermore, reducing to a point such regions would be equivalent to a pointlike localization of the constituent particles with obvious disastrous consequences.

The theories sketched above are nonrelativistic. The difficulty met in constructing a relativistic generalization is that the spherical regions mentioned above are not Lorentz invariant. Nevertheless, relativistic spontaneous localization models have been proposed. In the first model[14,15] a pointlike process acts on a light boson field which in turn is coupled to the fields describing the common material particles. In such a way the material particles are localized within regions whose dimensions are ruled by the light mass of the boson field. A pointlike process is nevertheless there, and this causes, e.g., an infinite rate of energy production. A second and a third

model[16] cure this problem partially and completely, respectively. We find it somewhat difficult, however, to grasp the physical meaning of the assumptions underlying the models.

In our opinion, compelling certain macroscopic quantities to have definite values is the essential feature of reduction theories. We shall try in the following to found the construction of a relativistic spontaneous localization model on the identification of a suitable set of macroscopic quantities to be induced by a stochastic process to have definite values.

In sect. 2 we describe the class of Markov processes in Hilbert space which induce the state vector to move towards the eigenspaces of the operator representing a quantity (or the common eigenspaces of the operators representing a set of compatible quantities). In sect. 3 the most reliable nonrelativistic reduction theory is concisely presented. In sects. 4 and 5 the framework for a relativistic reduction model is introduced. In sects. 6 and 7 we put forward a proposal for the quantities to be induced to have definite values and discuss it. In the final section we list some open problems and conclude.

2. Markov processes in Hilbert space

Given a Hamiltonian operator H and a selfadjoint linear operator A, let us consider the (Itô) stochastic differential equation

$$\mathrm{d}|\psi(t)\rangle = \left[-\frac{i}{\hbar}\,H\,\mathrm{d}t + g\,A_{\psi(t)}\mathrm{d}B(t) - \tfrac{1}{2}\,g^2\big(A_{\psi(t)}\big)^2\mathrm{d}t\right]|\psi(t)\rangle, \tag{2.1}$$

where A_ψ is the nonlinear operator

$$A_\psi = A - \langle\psi|A|\psi\rangle \tag{2.2}$$

and $B(t)$ is a Wiener process such that

$$\overline{\mathrm{d}B(t)} = 0, \qquad \overline{(\mathrm{d}B(t))^2} = \mathrm{d}t. \tag{2.3}$$

Eq. (2.1) conserves the norm of the state vector $|\psi\rangle$.

One can prove[11] that, if the Schrödinger term is dropped, the solutions of the resulting equation

$$\mathrm{d}|\psi(t)\rangle = \Big[g\,A_{\psi(t)}\mathrm{d}B(t) - \tfrac{1}{2}\,g^2\big(A_{\psi(t)}\big)^2\mathrm{d}t\Big]|\psi(t)\rangle \tag{2.4}$$

have the limit

$$|\psi(t)\rangle \xrightarrow[t\to\infty]{} P_\varepsilon|\psi(t_0)\rangle/\|P_\varepsilon\psi(t_0)\|, \qquad \Pr(\varepsilon) = \|P_\varepsilon\psi(t_0)\|^2, \tag{2.5}$$

where P_ε are the projection operators on the eigenspaces of A (with obvious modifications in the case of continuous spectrum). It is seen that the stochastic term drives $|\psi\rangle$ towards an eigenstate of A; or, in other words, it compels in the long run the quantity described by A to have a definite value. The process becomes ineffective when $|\psi(t)\rangle$ reaches an eigenvector of A because A_ψ is zero when applied to such a vector. At any time t_0, the probability of ending up in a definite eigenspace is the square norm of the projection of the state vector at time t_0 on that eigenspace and, in the case of degeneracy, the precise final eigenvector is given by the projection rule.

If both the Schrödinger term and the stochastic term are kept, the net result will depend on the competition, if it is there, between the two evolution processes.

If, instead of a single A, a set of commuting selfadjoint operators A^i is considered, the single stochastic term built with the operator A is replaced by one term for each A^i and the stochastic evolution equation becomes

$$\mathrm{d}|\psi(t)\rangle = \left[-\frac{i}{\hbar}\,H\,\mathrm{d}t + \sum\nolimits_i \left(g_i\,A^i_{\psi(t)}\,\mathrm{d}B_i(t) - \tfrac{1}{2}\,g_i^2\,\left(A^i_{\psi(t)}\right)^2\mathrm{d}t\right)\right]|\psi(t)\rangle, \tag{2.6}$$

where

$$\overline{\mathrm{d}B_i(t)} = 0, \qquad \overline{\mathrm{d}B_i(t)\mathrm{d}B_j(t)} = \delta_{ij}\mathrm{d}t. \tag{2.7}$$

If the Schrödinger term is dropped, the limit of the solutions is still given by eq. (2.5) where P_ε are now the projection operators on the common eigenspaces of the operators A^i.

3. The nonrelativistic mass process

The quantities being driven by the stochastic process to have definite values are the mass densities $D(\boldsymbol{x})$ averaged over small macroscopic spherical domains around all space points $\boldsymbol{x}$[13]. The evolution equation for the state vector $|\psi(t)\rangle$ is then

$$\mathrm{d}|\psi(t)\rangle = \left[-\frac{i}{\hbar}\,H\,\mathrm{d}t + \int\mathrm{d}^3\boldsymbol{x}\,\left(\frac{g_0}{m_0}\,D_{\psi(t)}(\boldsymbol{x})\,\mathrm{d}B_{\boldsymbol{x}}(t) - \tfrac{1}{2}\,\left(\frac{g_0}{m_0}\right)^2\left(D_{\psi(t)}(\boldsymbol{x})\right)^2\mathrm{d}t\right)\right]|\psi(t)\rangle, \tag{3.1}$$

where

$$D_{\psi(t)}(\boldsymbol{x}) = D(\boldsymbol{x}) - \langle\psi(t)|D(\boldsymbol{x})|\psi(t)\rangle \tag{3.2}$$

and the stochastic field $B_{\boldsymbol{x}}(t)$ has the properties

$$\overline{\mathrm{d}B_{\boldsymbol{x}}(t)} = 0, \qquad \overline{\mathrm{d}B_{\boldsymbol{x}}(t)\,\mathrm{d}B_{\boldsymbol{x}'}(t)} = \delta^{(3)}(\boldsymbol{x}-\boldsymbol{x}')\,\mathrm{d}t. \tag{3.3}$$

The strength constant g_0 is defined with respect to a reference mass m_0.

The macroscopic mass densities $D(\boldsymbol{x})$ are defined by

$$D(\boldsymbol{x}) = \int\mathrm{d}^3\bar{\boldsymbol{x}}\,F(\bar{\boldsymbol{x}}-\boldsymbol{x})\,m(\bar{\boldsymbol{x}}) \tag{3.4}$$

in terms of the microscopic mass density $m(\boldsymbol{x})$. The function $F(\boldsymbol{x})$ identifies the spherical domain over which the mass densities are averaged. It can be chosen to be the smooth function

$$F(\boldsymbol{x}) = \mathcal{N}\exp\left(-\tfrac{1}{2}\left(\boldsymbol{x}/a\right)^2\right), \qquad \mathcal{N} = \left(\frac{1}{2\pi a^2}\right)^{\frac{3}{2}}, \tag{3.5}$$

where the length constant a defines the linear dimensions of the small macroscopic domain. The microscopic mass density is

$$m(\boldsymbol{x}) = \sum\nolimits_k m_k \sum\nolimits_s a_k^+(\boldsymbol{x},s)\,a_k(\boldsymbol{x},s) \tag{3.6}$$

where k runs over the different kinds of identical particles and, for each k, m_k is the mass of the particles, $a_k^+(\boldsymbol{x},s)$, $a_k(\boldsymbol{x},s)$ are the creation and annihilation operators of particles at $\boldsymbol{x}$ with spin component s.

We remark that shrinking to a point the spherical domain which defines densities (i.e. using $m(\boldsymbol{x})$ instead of $D(\boldsymbol{x})$) is not permitted, not just because the consequences are unacceptable, but, more deeply, because the quantities driven to have a definite value must have a macroscopic

meaning. On the other hand, the spherical domain can neither be chosen too large, because the set of densities $D(\boldsymbol{x})$ would lose its power of distinguishing macroscopically distinguishable situations.

The suggested values of the parameters a and g_0 are

$$a \approx 10^{-5}\,\mathrm{cm}, \qquad g_0^2 \approx 10^{-30}\,\mathrm{cm}^3\,\mathrm{s}^{-1}, \tag{3.7}$$

having taken the proton mass as the reference mass m_0. It has been shown[11–13] that with such a choice superpositions of macroscopically distinguishable states are quickly reduced, while for the rest one has only negligible effects. Obviously, the theory is not exactly equivalent to standard quantum mechanics and is in principle falsifiable.

The precise form of the function $F(\boldsymbol{x})$ is practically unimportant. An alternative choice is the square function

$$F(\boldsymbol{x}) = \mathcal{N}\,\chi_{(0,a^2)}\left(\boldsymbol{x}^2\right), \qquad \mathcal{N} = \frac{3}{4\pi a^3}, \tag{3.8}$$

where $\chi_E(x)$ is the characteristic function of the set E.

Instead of using the mass densities averaged over the considered spherical volumes, one can equivalently express the process in terms of the total masses contained in the same volumes. Then the evolution equation becomes

$$\begin{aligned} \mathrm{d}|\psi(t)\rangle = \Bigg[& -\frac{i}{\hbar}\,H\,\mathrm{d}t \\ & + \int \mathrm{d}^3\boldsymbol{x}\left(g\left(M_{\psi(t)}(\boldsymbol{x})/m_0\right)\mathrm{d}B_{\boldsymbol{x}}(t) - \tfrac{1}{2}\,g^2\left(M_{\psi(t)}(\boldsymbol{x})/m_0\right)^2\mathrm{d}t\right)\Bigg]|\psi(t)\rangle, \end{aligned} \tag{3.9}$$

where

$$\begin{aligned} M_{\psi(t)}(\boldsymbol{x}) &= M(\boldsymbol{x}) - \langle\psi(t)|M(\boldsymbol{x})|\psi(t)\rangle, \\ M(\boldsymbol{x}) &= \int \mathrm{d}^3\bar{\boldsymbol{x}}\,\chi_{(0,a^2)}\left((\bar{\boldsymbol{x}}-\boldsymbol{x})^2\right)\,m(\bar{\boldsymbol{x}})\,. \end{aligned} \tag{3.10}$$

The squared strength constant g^2 is

$$g^2 = \left(\frac{3}{4\pi a^3}\right)^2 g_0^2 \approx 6\ 10^{-2}\,\mathrm{cm}^{-3}\,\mathrm{s}^{-1}. \tag{3.11}$$

4. The Tomonaga–Schwinger equation

In order to write down a Lorentz–invariant version of eq. (3.9), it is convenient to start from a manifestly covariant form of the Schrödinger equation. This is provided by the Tomonaga–Schwinger interaction–picture (IP) equation.

To simplify the notation from now on we shall use $\hbar = c = 1$ units.

In the Tomonaga–Schwinger approach the time t corresponding to the flat surface $t = \mathrm{itsvalue}$ is replaced by the general spacelike surface σ. The state vector is then a function $|\psi(\sigma)\rangle$ of σ. Denoting IP state vectors and operators by a superscript I, the Tomonaga–Schwinger evolution equation of the state vector is

$$\delta|\psi^{\mathrm{I}}(\sigma)\rangle = -i\,\mathcal{H}_{\mathrm{I}}^{\mathrm{I}}(x)|\psi^{\mathrm{I}}(\sigma)\rangle\delta\sigma(x) \tag{4.1}$$

where the spacetime point x belongs to σ, $\delta|\psi^{\mathrm{I}}(\sigma)\rangle$ is the change of the state vector in going from σ to a nearby spacelike surface separated from σ by an arbitrarily small fourdimensional

bubble adjacent to x, $\delta\sigma(x)$ is the spacetime volume of the bubble, and $\mathcal{H}^{\rm I}_{\rm I}(x)$ is the density of IP interaction Hamiltonian at x. If all couplings among different fields are nonderivative, $\mathcal{H}^{\rm I}_{\rm I}(x)$ is automatically a Lorentz scalar. If derivative couplings are there, a somewhat more complicated construction is necessary and the role of density of interaction Hamiltonian is played by a scalar built with the part of the energy–momentum tensor corresponding to the interaction Lagrangian and the orientation of the element of σ surrounding x. Eq. (4.1) is then Lorentz invariant in all cases.

Since one can go from a spacelike surface to another one following different paths in the manifold of spacelike surfaces, the evolution calculated using eq. (4.1) must be independent of such different paths. The fulfilment of this condition of integrability follows from the commutativity of Hamiltonian densities at different spacelike–separated points.

In a particular reference frame, one can choose the surfaces σ to be the flat surfaces corresponding to arbitrary values of t and investigate the change $\mathrm{d}|\psi^{\rm I}(t)\rangle$ of the IP state vector in going from t to $t+\mathrm{d}t$. Let $C(\boldsymbol{x}_i)$ be a set of disjoint arbitrarily small cubes exhausting the tridimensional space. If $\mathrm{d}V$ is the volume of the cubes, the volume of each spacetime domain identified by a cube and by the time interval $(t, t+\mathrm{d}t)$ is

$$\delta\sigma(t,\boldsymbol{x}_i) = \mathrm{d}V\,\mathrm{d}t. \tag{4.2}$$

Then, according to eq. (4.1),

$$\mathrm{d}|\psi^{\rm I}(t)\rangle = -i\sum\nolimits_i \mathcal{H}^{\rm I}_{\rm I}(t,\boldsymbol{x}_i)\mathrm{d}V\,|\psi^{\rm I}(t)\rangle\,\mathrm{d}t = -i\,H^{\rm I}_{\rm I}(t)|\psi^{\rm I}(t)\rangle\,\mathrm{d}t, \tag{4.3}$$

where $H^{\rm I}_{\rm I}(t)$ is the IP interaction Hamiltonian operator in the considered reference frame.

5. Relativistic reduction equation

In a Tomonaga–Schwinger framework, we propose the stochastic evolution equation

$$\delta|\psi^{\rm I}(\sigma)\rangle = \left[-i\,\mathcal{H}^{\rm I}_{\rm I}(x)\,\delta\sigma(x) + g\,S^{\rm I}_{\psi(\sigma)}(x)\,\delta\beta(x) - \tfrac{1}{2}\,g^2\Big(S^{\rm I}_{\psi(\sigma)}(x)\Big)^2\delta\sigma(x)\right]|\psi^{\rm I}(\sigma)\rangle, \tag{5.1}$$

where

$$S^{\rm I}_{\psi(\sigma)}(x) = S^{\rm I}(x) - \langle\psi^{\rm I}(\sigma)|S^{\rm I}(x)|\psi^{\rm I}(\sigma)\rangle \tag{5.2}$$

and, in correspondence to the spacetime bubble $\delta\sigma(x)$, $\delta\beta(x)$ is a Gaussian random variable such that

$$\overline{\delta\beta(x)} = 0, \qquad \overline{\big(\delta\beta(x)\big)^2} = \delta\sigma(x). \tag{5.3}$$

The linear operator $S^{\rm I}(x)$ is intended to be a Lorentz scalar field describing a macroscopic quantity associated in a proper way to the point x. If $S^{\rm I}(x)$ is a scalar, the Lorentz invariance of eq. (5.1) is obvious.

When the evolution is calculated, a collection x_i of spacetime points is considered together with the spacetime bubbles $\delta\sigma(x_i)$ adjacent to them. It is understood that the corresponding random variables $\delta\beta(x_i)$ are independent, so that

$$\overline{\delta\beta(x_i)} = 0, \qquad \overline{\delta\beta(x_i)\,\delta\beta(x_j)} = \delta_{ij}\,\delta\sigma(x_i). \tag{5.4}$$

Eq. (5.4) is compatible with the arbitrary smallness of the spacetime bubbles. In fact, given a collection of bubbles $\delta\sigma(x_i)$, let $\delta\sigma(x_{ik})$ be, for each i and for running k, a decomposition in parts of $\delta\sigma(x_i)$, so that, in terms of volumes,

$$\delta\sigma(x_i) = \sum\nolimits_k \delta\sigma(x_{ik}). \tag{5.5}$$

Let $\delta\beta(x_{ik})$ be the Gaussian random variables corresponding to the finer collection of bubbles, such that

$$\overline{\delta\beta(x_{ik})} = 0, \qquad \overline{\delta\beta(x_{ik})\,\delta\beta(x_{jl})} = \delta_{ij}\,\delta_{kl}\,\delta\sigma(x_{ik}). \tag{5.6}$$

Then

$$\delta\beta(x_i) \equiv \sum\nolimits_k \delta\beta(x_{ik}) \tag{5.7}$$

are Gaussian random variables which, because of eqs. (5.6) and (5.5) satisfy condition (5.4). Therefore, to each realization of the random variables $\delta\beta(x_{ik})$ there corresponds a realization of the random variables $\delta\beta(x_i)$ and the statistical distributions of such realizations agree.

Similarly to the pure Tomonaga–Schwinger equation, the integrability of eq. (5.1) has to be demonstrated. We postpone the discussion of this problem. Assuming integrability, it follows from eq. (5.4) that the change $\delta_{\{i\}}|\psi^{\mathrm{I}}(\sigma)\rangle$ of the state vector in going from σ to a nearby surface separated from σ by a collection of bubbles $\delta\sigma(x_i)$ is given by

$$\begin{aligned}\delta_{\{i\}}|\psi^{\mathrm{I}}(\sigma)\rangle = \Big[&- i \sum\nolimits_i \mathcal{H}^{\mathrm{I}}_{\mathrm{I}}(x_i)\delta\sigma(x_i) \\ &+ \sum\nolimits_i \Big(g\, S^{\mathrm{I}}_{\psi(\sigma)}(x_i)\,\delta\beta(x_i) - \tfrac{1}{2}\, g^2 \Big(S^{\mathrm{I}}_{\psi(\sigma)}(x_i)\Big)^2 \delta\sigma(x_i)\Big)\Big] |\psi^{\mathrm{I}}(\sigma)\rangle.\end{aligned} \tag{5.8}$$

In a particular reference frame, we again choose the surfaces σ to be the flat surfaces corresponding to arbitrary values of t and investigate the change $\mathrm{d}|\psi^{\mathrm{I}}(t)\rangle$ of the state vector in going from t to $t + \mathrm{d}t$. In the considered reference frame, we envisage a stochastic field $B_{\boldsymbol{x}}(t)$ such that

$$\delta\beta(t, \boldsymbol{x}_i) = \int_{C(\boldsymbol{x}_i)} \mathrm{d}^3\boldsymbol{x}\ \mathrm{d}B_{\boldsymbol{x}}(t), \tag{5.9}$$

where $C(\boldsymbol{x}_i)$ are cubes as those of sect. 4. It is easily checked that the properties

$$\overline{\mathrm{d}B_{\boldsymbol{x}}(t)} = 0, \qquad \overline{\mathrm{d}B_{\boldsymbol{x}}(t)\,\mathrm{d}B_{\boldsymbol{x}'}(t)} = \delta^{(3)}(\boldsymbol{x} - \boldsymbol{x}')\,\mathrm{d}t \tag{5.10}$$

imply, through relations (5.9) and (4.2), eq. (5.4). In fact

$$\begin{aligned}\overline{\delta\beta(t, \boldsymbol{x}_i)} &= 0, \\ \overline{\delta\beta(t, \boldsymbol{x}_i)\,\delta\beta(t, \boldsymbol{x}_j)} &= \int_{C(\boldsymbol{x}_i)} \mathrm{d}^3\boldsymbol{x} \int_{C(\boldsymbol{x}_j)} \mathrm{d}^3\boldsymbol{x}'\ \overline{\mathrm{d}B_{\boldsymbol{x}}(t)\,\mathrm{d}B_{\boldsymbol{x}'}(t)} \\ &= \int_{C(\boldsymbol{x}_i)} \mathrm{d}^3\boldsymbol{x} \int_{C(\boldsymbol{x}_j)} \mathrm{d}^3\boldsymbol{x}'\ \delta^3(\boldsymbol{x} - \boldsymbol{x}')\,\mathrm{d}t = \delta_{ij}\,\mathrm{d}V\mathrm{d}t = \delta_{ij}\,\delta\sigma(t, \boldsymbol{x}_i)\,.\end{aligned} \tag{5.11}$$

Then, applying eq. (5.8) to the calculation of $\mathrm{d}|\psi^{\mathrm{I}}(t)\rangle$, one finds

$$\begin{aligned}\mathrm{d}|\psi^{\mathrm{I}}(t)\rangle &= \Big[- i \sum\nolimits_i \mathcal{H}^{\mathrm{I}}_{\mathrm{I}}(t, \boldsymbol{x}_i)\mathrm{d}V\,\mathrm{d}t \\ &\qquad + \sum\nolimits_i \Big(g\, S^{\mathrm{I}}_{\psi(t)}(t, \boldsymbol{x}_i) \int_{C(\boldsymbol{x}_i)} \mathrm{d}^3\boldsymbol{x}\ \mathrm{d}B_{\boldsymbol{x}}(t) - \tfrac{1}{2}\, g^2 \big(S^{\mathrm{I}}_{\psi(t)}(t, \boldsymbol{x}_i)\big)^2 \mathrm{d}V\,\mathrm{d}t\Big)\Big] |\psi^{\mathrm{I}}(t)\rangle \\ &= \Big[- i\, H^{\mathrm{I}}_{\mathrm{I}}(t)\,\mathrm{d}t + \int \mathrm{d}^3\boldsymbol{x} \Big(g\, S^{\mathrm{I}}_{\psi(t)}(t, \boldsymbol{x})\,\mathrm{d}B_{\boldsymbol{x}}(t) - \tfrac{1}{2}\, g^2 \big(S^{\mathrm{I}}_{\psi(t)}(t, \boldsymbol{x})\big)^2 \mathrm{d}t\Big)\Big] |\psi^{\mathrm{I}}(t)\rangle.\end{aligned} \tag{5.12}$$

Eq. (5.12) is written in the interaction picture. We note, however, that the relationship between the interaction picture and the Schrödinger picture is much less trivial in the situation we are interested in than in the common applications of quantum field theory. In fact, in our case, bound subsystems of macroscopic systems must necessarily be considered and, on the other hand, the free evolution operator $U_0(t)$ which connects the two pictures destroys any bound state because it contains no interaction, so that it simply moves each constituent particle according to its momentum content in the bound state. As a consequence, describing bound states in the interaction picture is practically impossible. For this reason it is convenient to rewrite eq. (5.12) in the Schrödinger picture (SP). Denoting state vectors and operators in the SP by no superscript, the two pictures are related by

$$|\psi(t)\rangle = U_0(t)\,|\psi^{\mathrm{I}}(t)\rangle, \qquad A = U_0(t)\,A^{\mathrm{I}}(t)\,U_0^+(t) \tag{5.13}$$

and the evolution equation for $|\psi(t)\rangle$ turns out to be

$$\mathrm{d}|\psi(t)\rangle = \Big[-i\,H\mathrm{d}t + \int \mathrm{d}^3\boldsymbol{x}\,\Big(g\,S_{\psi(t)}(\boldsymbol{x})\,\mathrm{d}B_{\boldsymbol{x}}(t) - \tfrac{1}{2}\,g^2\big(S_{\psi(t)}(\boldsymbol{x})\big)^2\mathrm{d}t\Big)\Big]\,|\psi(t)\rangle, \tag{5.14}$$

where H is the total Hamiltonian operator and

$$S_{\psi(t)}(\boldsymbol{x}) = S(\boldsymbol{x}) - \langle\psi(t)|S(\boldsymbol{x})|\psi(t)\rangle, \tag{5.15}$$

$$S(\boldsymbol{x}) = U_0(t)\,S^{\mathrm{I}}(t,\boldsymbol{x})\,U_0^+(t). \tag{5.16}$$

We note the formal similarity of eq. (5.14) to eqs. (3.1) or (3.9). Eq. (5.14) is not manifestly Lorentz invariant. It is, however, Lorentz invariant provided the operator $S^{\mathrm{I}}(x)$ appearing in eq. (5.16) is a Lorentz scalar. Of course, each element of eq. (5.14) must be properly transformed in going from a reference frame to another one.

6. Stuff

The macroscopic operators $S^{\mathrm{I}}(x)$ are IP operators defined by

$$S^{\mathrm{I}}(x) = \int_{D(x)} \mathrm{d}^4\bar{x}\,s^{\mathrm{I}}(\bar{x}), \tag{6.1}$$

where $s^{\mathrm{I}}(x)$ is a Lorentz scalar field built with IP field operators at spacetime point x and their derivatives. The integration domain $D(x)$ (Fig. 1) is the set of points $\bar{x} \equiv (\bar{t}, \bar{\boldsymbol{x}})$ such that

$$-a^2 \le (\bar{t} - t)^2 - (\bar{\boldsymbol{x}} - \boldsymbol{x})^2 \le a^2, \tag{6.2}$$

where a is a small macroscopic length which is intended to play the same role played by a in the nonrelativistic theory of sect. 3. Clearly, $S^{\mathrm{I}}(x)$ is in turn a Lorentz scalar field. Other choices are possible for the domain $D(x)$, but we consider here only the definition (6.2). The microscopic operators $s^{\mathrm{I}}(x)$ will be defined below as representing the spatial density of a quantity we call stuff which is related to the presence of massive particles. Therefore the quantity represented by $S^{\mathrm{I}}(x)$ has the meaning of a time–integrated amount of stuff.

The domain $D(x)$ extends to infinity in spacetime. Therefore the states $|\psi^{\mathrm{I}}(\sigma)\rangle$, $|\psi^{\mathrm{I}}(t)\rangle$ or $|\psi(t)\rangle$ must in principle be intended as states of the universe. Then the main problem with the set of quantities (6.1) is that they could be unable to distinguish locally different distributions of stuff because of the overwhelming contributions to them of remote stuff. We shall see in sect. 7 that this problem is not there.

We propose that the density of stuff is the quantum analogue of the classical quantity

$$s^{\mathrm{C}}(\bar{x}) = T^{\mu}{}_{\mu}(\bar{x}), \tag{6.3}$$

i.e. the invariant trace of the energy–momentum tensor $T^{\mu\nu}(\bar{x})$. Since dimensionally $T^{\mu}{}_{\mu}(\bar{x})$ is a density of energy, the time–integrated amount of stuff is then an action.

To illustrate the physical meaning of $T^{\mu}{}_{\mu}(\bar{x})$ and of the ensuing time–integrated amount of stuff we make reference to the case of classical mechanical systems. For a free pointlike particle moving with velocity $\boldsymbol{v}$ one finds

$$T^{\mu}{}_{\mu}(\bar{x}) = m\sqrt{1-\boldsymbol{v}^2}\,\delta^{(3)}(\bar{\boldsymbol{x}} - \boldsymbol{x}_0 - \boldsymbol{v}\bar{t}), \tag{6.4}$$

m being the rest mass of the particle. The corresponding value of the time–integrated amount of stuff is

$$\begin{aligned} S^{C}(x) &= \int_{D(x)} \mathrm{d}^4\bar{x}\, s^{\mathrm{C}}(\bar{x}) \\ &= 2m\left(\sqrt{\boldsymbol{x}_1^2 + (\boldsymbol{x}_1\cdot\boldsymbol{v})^2/(1-\boldsymbol{v}^2) + a^2} - \sqrt{\boldsymbol{x}_1^2 + (\boldsymbol{x}_1\cdot\boldsymbol{v})^2/(1-\boldsymbol{v}^2) - a^2}\right), \end{aligned} \tag{6.5}$$

where $\boldsymbol{x}_1 = \boldsymbol{x}_0 + \boldsymbol{v}t - \boldsymbol{x}$ is the position of the particle at time t referred to $\boldsymbol{x}$ and the second square root disappears if its argument is negative. If the particle is in $\boldsymbol{x}$ at time t, then $\boldsymbol{x}_1 = 0$ and

$$S^{C}(x) = 2ma. \tag{6.6}$$

If the particle is far from $\boldsymbol{x}$, i.e. $|\boldsymbol{x}_1|$ is large, we consider two extreme situations. First, let the trajectory of the particle be orthogonal to $\boldsymbol{x}_1$. Then $\boldsymbol{x}_1\cdot\boldsymbol{v} = 0$ and, for large $|\boldsymbol{x}_1|$, we find

$$S^{C}(x) = 2ma\,\frac{a}{|\boldsymbol{x}_1|}\cdot \tag{6.7}$$

On the opposite, if the trajectory is parallel to $\boldsymbol{x}_1$, then $(\boldsymbol{x}_1\cdot\boldsymbol{v})^2 = \boldsymbol{x}_1^2\boldsymbol{v}^2$ and, for large $|\boldsymbol{x}_1|$, we get

$$S^{C}(x) = 2ma\,\sqrt{1-\boldsymbol{v}^2}\,\frac{a}{|\boldsymbol{x}_1|}\cdot \tag{6.8}$$

In both the extreme cases the time–integrated amount of stuff of a far particle is significantly reduced with respect to that of a particle in $\boldsymbol{x}$ and we expect that a similar conclusion holds also in the intermediate situations.

If several free particles are present, a similar contribution from each particle is there. In the case of a noninteracting distribution of mass at rest in the considered reference frame, $T^{\mu}{}_{\mu}(\bar{x})$ is the distribution of mass itself and the time–integrated amount of stuff is given by an integral in tridimensional space where each volume element is multiplied by the density of mass in that element times the extension of the time interval (or intervals) allowed by inequalities (6.2) in correspondence with that element. The farther is the element of volume from $\boldsymbol{x}$, the smaller is such a time extension.

That we have found is just the type of behaviour we need in order that the quantities $S^{C}(x)$ for suitable values of x be able to distiguish among different distributions of stuff.

The above examples concern noninteracting systems. If interactions are there, they will also contribute to $T^{\mu}{}_{\mu}(\bar{x})$ and $S^{C}(x)$.

In the case of a field the energy–momentum tensor is

$$T^{\mu\nu} = \sum\nolimits_{\kappa} \partial^{\mu}\phi^{\kappa} \frac{\partial \mathcal{L}}{\partial_{\nu}\phi^{\kappa}} - g^{\mu\nu}\mathcal{L} + \partial_{\lambda} A^{\mu\nu\lambda}, \tag{6.9}$$

where $\mathcal{L}$ is the Lagrangian density, κ runs over all components of the field ϕ and $A^{\mu\nu\lambda}$ is an arbitrary tensor antisymmetric in the last two indices to be chosen so that $T^{\mu\nu}$ is symmetric. Except for the electromagnetic field, classical relativistic fields have no direct physical meaning. One should go to the corresponding quantum fields with their particle interpretation and, in such a framework, undertake the description of the world. For the electromagnetic field, both classical and quantum, $T^{\mu}{}_{\mu}$ and consequently the time–integrated amount of stuff are identically zero. The other cases of interest are presently being investigated. If the meaning and behaviour of the time–integrated amount of stuff as it emerges from the above classical mechanical examples will be confirmed, we think that $T^{\mu}{}_{\mu}$ is a good candidate for the density of stuff.

7. Stuff operators

When interactions are present, and of course they actually are there, the quantum analogues of $s^{\mathrm{C}}(\bar{x})$ and $S^{C}(x)$ are the corresponding Heisenberg–picture operators, not the interaction–picture operators $s^{\mathrm{I}}(\bar{x})$ and $S^{\mathrm{I}}(x)$ appearing in and defined by eq. (6.1). The difference does not consist simply in the inclusion in the density of stuff $T^{\mu}{}_{\mu}(\bar{x})$ of the contributions coming from the interactions, but lies in the fact that defining the time–integrated amount–of–stuff operator in one picture of time evolution or another is not the same thing. This inequivalence is a consequence of the time integration present in the definition (6.1) and such a time integration is necessary if $S^{\mathrm{I}}(x)$ has to be a Lorentz scalar.

Choosing a reference frame and identifying σ with time t, let us write down explicitly the Schrödinger–picture operator $S(\boldsymbol{x})$ related to $S^{\mathrm{I}}(x)$ by eq. (5.16). From

$$S^{\mathrm{I}}(t, \boldsymbol{x}) = \int_{-\infty}^{+\infty} \mathrm{d}\bar{t} \int_{D_{\bar{t}}(t,\boldsymbol{x})} \mathrm{d}^3\bar{\boldsymbol{x}}\, s^{\mathrm{I}}(\bar{t}, \bar{\boldsymbol{x}}), \tag{7.1}$$

where $D_{\bar{t}}(t, \boldsymbol{x})$ is the sphere or the spherical shell of points $\bar{\boldsymbol{x}}$ satisfying condition (6.2) for fixed $\bar{t}$, denoting again by no superscript SP operators one gets

$$\begin{aligned} S(\boldsymbol{x}) &= U_0(t) \int_{-\infty}^{+\infty} \mathrm{d}\bar{t} \int_{D_{\bar{t}}(t,\boldsymbol{x})} \mathrm{d}^3\bar{\boldsymbol{x}}\, U_0^{+}(\bar{t})\, s(\bar{\boldsymbol{x}})\, U_0(\bar{t})\, U_0^{+}(t) \\ &= \int_{-\infty}^{+\infty} \mathrm{d}\bar{t}\, U_0^{+}(\bar{t} - t) \int_{D_{\bar{t}}(t,\boldsymbol{x})} \mathrm{d}^3\bar{\boldsymbol{x}}\, s(\bar{\boldsymbol{x}})\, U_0(\bar{t} - t). \end{aligned} \tag{7.2}$$

It is easily checked that $S(\boldsymbol{x})$ is actually independent of t. If we should have written eq. (6.1) in the Heisenberg picture, the full time–evolution operator U would appear in eq. (7.2) instead of U_0.

It is seen that when the operator $S(\boldsymbol{x})$ is applied to the SP state $|\psi(t)\rangle$ one must evolve freely the state from time t to time $\bar{t}$, apply the SP operator having the meaning of amount of stuff contained in the domain $D_{\bar{t}}(t, \boldsymbol{x})$ and then evolve back the result, again freely, up to the time t. Finally one integrates over time $\bar{t}$.

We must now understand whether the set of operators $S(\boldsymbol{x})$ does the job we want it do.

We first consider a local system, i.e. a system contained in a finite spatial domain, the system's environment being forgotten. Let the SP state at time t be

$$|\psi(t)\rangle = \sum\nolimits_{\alpha} c_{\alpha} |\lambda_{\alpha}(t)\rangle, \tag{7.3}$$

where the states $|\lambda_\alpha(t)\rangle$ are macroscopically distinguishable on the basis of their different macroscopic distributions of stuff.

To allow for a simple description of what is going on, we assume that the system consists of a single macroscopic object. Each term in expansion (7.3) describes a unique bound system whose center of mass is in a definite state different for different terms. When the free time–evolution operator $U_0(\bar{t}-t)$ is applied, this structure is maintained in the sense that the center of mass remains in a definite state and moves accordingly. However, the object as a bound state is shattered by U_0, as we have already discussed in sect. 5. In order to estimate the result of the application of the operator (7.2) to the different terms in expansion (7.3), we suppose that the envisaged object is at rest in the considered reference frame and in two different positions for, say, $\alpha = 1$ and $\alpha = 2$ (Fig. 2). Some amount of stuff $\mathcal{S}$ belonging to the object lies at $\boldsymbol{x}$ for $\alpha = 1$ and at a distance l from $\boldsymbol{x}$ for $\alpha = 2$. Let the amount of stuff $\mathcal{S}$ be projected around isotropically with velocity w by the operator $U_0(\bar{t}-t)$. Then the time–integrated amount of stuff counted by the operator (7.2) is easily estimated and turns out to be

$$\begin{array}{cc} 2a\mathcal{S}\left[\dfrac{1}{\sqrt{1-w^2}}\right], & 2a\mathcal{S}\,\dfrac{a}{l}\left[\dfrac{2}{\pi}\,K\big(w^2\big)\right], \\ (\alpha=1) & (\alpha=2) \end{array} \tag{7.4}$$

where the expression for the case $\alpha = 2$ is evaluated for $l^2 \gg a^2$ and the function $K(\,\cdot\,)$ is the complete elliptic integral of the first kind. The factor in square brackets is always smaller for $\alpha = 2$ than for $\alpha = 1$. Therefore, in the case of the considered example, the set of operators (7.2) discriminates the various terms in expansion (7.3). If the object is moving, and $\mathcal{S}$ with it, the above argument is to be supplemented by the discussion in sect. 6, the particle considered there being identified with the center of mass of $\mathcal{S}$. We think it is evident that our conclusion remains true for general superpositions of macroscopically distinguishable states.

As already remarked, the states in our equations are in principle states of the universe, so that, besides the local system, we must take into account the rest of the universe, which we call the environment. For large $|\bar{t}-t|$ the domain $D_{\bar{t}}(t,\boldsymbol{x})$ is a spherical shell centered in $\boldsymbol{x}$ whose volume is $4\pi|\bar{t}-t|a^2$. The first problem which then arises concerns the convergence of the integrals in the definition of $S(\boldsymbol{x})$. This problem disappears if we assume that the universe is finite, but another problem remains. If we include the environment in the description, its contribution to the value of $S(\boldsymbol{x})$ will be overwhelmingly larger than the contribution of the local system, so that the values of $S(\boldsymbol{x})$ corresponding to macroscopically distinguishable states of the local system will no more differ significantly. As a consequence the stochastic process in eq. (5.14) could become ineffective.

To discuss the problem presented above, we assume that the splitting between local system and environment is such that the particles in the environment can be considered as distinguishable from the particles in the local system even when they are of the same kind. This does not mean that the particles in the two subsystems of the universe do not interact, it means that the dynamics is such that they never come together in space. We do not think that this assumption is essential, but it considerably simplifies the discussion which follows. If particles of the same kind in the two subsystems are distinguishable, they can be described by distinct fields and, as a consequence, the SP state of the universe can be written as

$$|\psi(t)\rangle = \sum\nolimits_\alpha c_\alpha\,|\lambda_\alpha(t)\rangle\,|\varepsilon_\alpha(t)\rangle, \tag{7.5}$$

where $|\lambda_\alpha(t)\rangle$ and $|\varepsilon_\alpha(t)\rangle$ indicate states of the local system and of the environment, respectively. The free evolution operator $U_0(t)$ is the product of an operator acting on the states $|\lambda_\alpha(t)\rangle$ and an operator acting on the states $|\varepsilon_\alpha(t)\rangle$. May be that some particles of the environment and some particles of the local system are brought to overlap by the free evolution operator $U_0(\bar{t}-t)$.

The SP microscopic density–of–stuff operator $s(\bar{\boldsymbol{x}})$ will contain terms coming from the free Lagrangian density of the fields and terms coming from the interaction. If the contribution of the interaction between local–system fields and environment fields is disregarded in $s(\bar{\boldsymbol{x}})$, then the SP time–integrated amount–of–stuff operator is the sum

$$S(\boldsymbol{x}) = L(\boldsymbol{x}) + E(\boldsymbol{x}), \tag{7.6}$$

of a term $L(\boldsymbol{x})$ acting on the local states $|\lambda_\alpha(t)\rangle$ and a term $E(\boldsymbol{x})$ acting on the environment states $|\varepsilon_\alpha(t)\rangle$. In the situation in which we are interested, the local states $|\lambda_\alpha(t)\rangle$ are distinguishable on the basis of their different macroscopic distributions of stuff. The environment states $|\varepsilon_\alpha(t)\rangle$ are mutually orthogonal if decoherence is there, as it is reasonable to assume. Nevertheless, they are macroscopically undistinguishable, so that

$$E(\boldsymbol{x})\,|\varepsilon_\alpha(t)\rangle = e(t,\boldsymbol{x})\,|\varepsilon_\alpha(t)\rangle, \tag{7.7}$$

with the eigenvalues $e(t,\boldsymbol{x})$ independent of α. It is then easily shown that the two contributions of $E(\boldsymbol{x})$ in eq. (5.15) cancel each other, so that

$$\begin{aligned}\Big(S(\boldsymbol{x}) - \langle\psi(t)|S(\boldsymbol{x})|\psi(t)\rangle\Big)|\psi(t)\rangle &= \Big(L(\boldsymbol{x}) - \sum\nolimits_\alpha |c_\alpha|^2 \langle\lambda_\alpha(t)|L(\boldsymbol{x})|\lambda_\alpha(t)\rangle\Big)\,|\psi(t)\rangle \\ &= \Big(L(\boldsymbol{x}) - \langle\psi(t)|L(\boldsymbol{x})|\psi(t)\rangle\Big)|\psi(t)\rangle,\end{aligned} \tag{7.8}$$

and similarly when the bracket is squared.

We conclude that the set of operators $S(\boldsymbol{x})$ is able to discriminate among states having locally different macroscopic distributions of stuff even when the states are states of the universe and, as a consequence, the stochastic term in eq. (5.14) causes the type of reduction we are interested in.

8. Open problems and conclusion

So far we took for granted that the time–integrated amount–of–stuff operators (6.1) describe classical macroscopic quantities. This means that they should mutually commute to a good approximation and that they should not exhibit appreciable fluctuations related to the microscopic structure of the system. This behaviour should be a consequence of the choice of the constant a, defining the integration domain $D(x)$, as a small but macroscopic length. We have few doubts that an operator like the field–theoretic $T^\mu{}_\mu(\bar{x})$ integrated over a sufficiently large spacetime domain possesses such properties. However, the space domain $D_{\bar{t}}(t,\boldsymbol{x})$ corresponding for given $\bar{t}$ to the spacetime domain $D(x)$ is, for large $|\bar{t}-t|$, a spherical shell of radius $|\bar{t}-t|$ and of vanishingly small thickness $a^2/|\bar{t}-t|$. We hope to be able to prove that this particular shape of the integration domain does not spoil the classical macroscopic features of the operators (6.1). We are presently working on this problem.

As mentioned in sect. 5, the result of the application of the modified Tomonaga–Schwinger equation (5.1) should be independent of the path in the manifold of spacelike surfaces followed in going from a spacelike surface to another one. We do not know whether there exists a general mathematical theory allowing to deal with such an integrability problem. In the case of the pure Tomonaga–Schwinger equation, the commutativity of the operators $\mathcal{H}^{\mathrm{I}}_{\mathrm{I}}(x)$ at different spacelike separated points is likely to be sufficient to ensure integrability. Similarly, if the Schrödinger term is dropped from eq. (5.1), the commutativity of operators $S^{\mathrm{I}}(x)$ related to their classical nature should be sufficient for integrability. If both the Schrödinger and the stochastic term are retained, however, the commutativity argument breaks down, because, considered $S^{\mathrm{I}}(x)$ and $\mathcal{H}^{\mathrm{I}}_{\mathrm{I}}(x')$, $S^{\mathrm{I}}(x)$ contains contributions from points which are not spacelike separated from

the point x' even if x and x' are spacelike separated. We note, however, that the stochastic process is effective only when superpositions of macroscopically distinguishable states are there and that, when such a kind of superposition starts to be created in the region around x', $S^{\mathrm{I}}(x)$ is not able (is little able) to distinguish among the superposed terms if x is away from x'. Therefore, advancing the spacelike surface first around x' and then around x or viceversa should be unimportant. We plan to discuss this point in model situations.

The stochastic equation (5.1) and the time–integrated amount–of–stuff operators (6.1) contain altogether two new constants, the length a and the strength constant g. As already said, a has to be a small macroscopic length. The same value 10^{-5}cm proposed for the nonrelativistic model is a reasonable choice. Suggesting a value for g is much more difficult, even though one could try to get a hint from the value of the strength constant for the nonrelativistic model, using the formal similarity of eq. (5.14) to eq. (3.9). Of course, the goal is again getting from the stochastic process a rapid suppression of superpositions of macroscopically distinguishable states and negligible effects for all the rest. The evaluation of the physical consequences of our model, depending on the value of g is a job to be done.

Finally, the interpretative implications of the type of theory we are proposing are to be investigated.

Concluding, we do not presently have definite and definitive answers to the problems presented above. In any case we are convinced that the right road to build a relativistic reduction theory is that of identifying a suitable set of macroscopic quantities to be stochastically compelled to have definite values, because this is, we think, the physical meaning of reduction.

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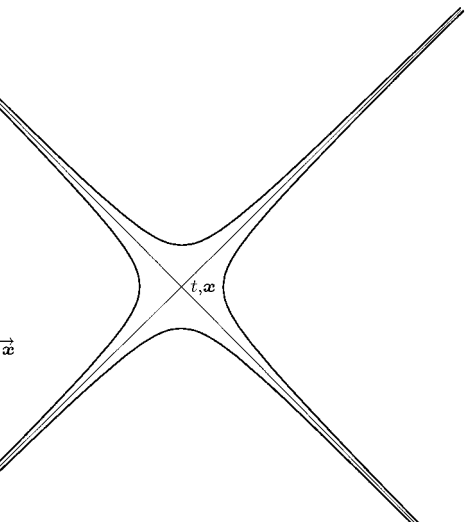
t,x
x

Fig. 1. The integration domain $D(x)$ represented in a bidimensional spacetime.

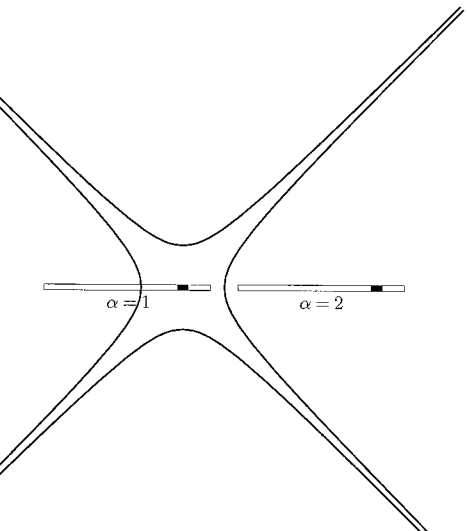
$\alpha = 1$
$\alpha = 2$

Fig. 2. A macroscopic object in two different positions. The thick dash represents an amount of stuff belonging to the object.